EBAU - INTEGRAL.

2021 OHIKOA - $\boxed{A4}$

$$I = \int \frac{7x+13}{(x+1)\underbrace{(x^2-x-2)}_{(x-2)(x+1)}}\,dx = \int \frac{7x+13}{(x+1)^2(x-2)}$$

$$\frac{7x+13}{(x+1)^2(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x-2)}$$

$$\frac{7x+13}{(x+1)^2(x-2)} = \frac{A(x+1)(x-2) + B(x-2) + C(x+1)^2}{(x+1)^2(x-2)}$$

$$7x+13 = A(x+1)(x-2) + B(x-2) + C(x+1)^2$$

$x=2 \longrightarrow 14+13 = C\cdot 9 \longrightarrow \boxed{C=3}$

$x=-1 \longrightarrow -7+13 = B(-3) \longrightarrow \boxed{B=-2}$

$x=0 \longrightarrow 13 = A\cdot 1(-2) + (-2)(-2) + 3\cdot 1^2$

$13 = -2A + 4 + 3$

$\longrightarrow \boxed{A=-3}$

$$\int \frac{7x+13}{(x+1)(x^2-x-2)} = \int \frac{-3}{x+1}\,dx + \underbrace{\int \frac{-2}{(x+1)^2}}_{-2(x+1)^{-2}}dx + \int \frac{3}{x-2}\,dx =$$

$$= -3\ln|x+1| - 2\,\frac{(x+1)^{-2+1}}{-2+1} + 3\ln|x-2| + K$$

$$\boxed{I = -3\ln|x+1| + 2\,\frac{1}{x+1} + 3\ln|x-2| + K}$$

2022 EZ OHIKOA

$\boxed{A4}$ $$\int \ln(x^2-1)\cdot dx = I$$

ZATIKAKO FUNTZIOA $\qquad \int u\,dv = u\cdot v - \int v\cdot du$

$$\begin{cases} u=\ln(x^2-1) \longrightarrow du=\dfrac{2x}{x^2-1}dx \\ dv=dx \longrightarrow v=dx \end{cases}$$

$$I=\ln(x^2-1)\cdot x-\int\frac{x\cdot 2x}{x^2-1}dx = x\cdot\ln(x^2-1)-2\int\frac{x^2}{x^2-1}dx$$

$$=x\cdot\ln(x^2-1)-2\int\left(1+\frac{1}{x^2-1}\right)dx=$$

$$\begin{array}{r|l} x^2 & x^2-1 \\ \hline -x^2+1 & 1 \\ 1 & \end{array}$$

$$=x\cdot\ln(x^2-1)-2\int dx-2\int\frac{1}{x^2-1}dx=$$

$$=x\cdot\ln(x^2-1)-2x-2\underbrace{\int\frac{1}{x^2-1}dx}_{I_2}$$

$$I_2=\int\frac{1}{x^2-1}dx \quad // \quad \frac{1}{x^2-1}=\frac{A}{x+1}+\frac{B}{x-1}$$

$$1=A(x-1)+B(x+1)$$

$$x=1\to \quad 1=2B\to \boxed{B=1/2}$$

$$x=-1\to \quad 1=-2A\to \boxed{A=-1/2}$$

$$I_2=\int\frac{1}{x^2-1}dx=\int\frac{-1/2}{x+1}dx+\int\frac{1/2}{x-1}dx=$$

$$=\boxed{-\frac{1}{2}\ln|x+1|+\frac{1}{2}\ln|x-1|}$$

$$\boxed{I=x\cdot\ln(x^2-1)-2x+\underbrace{\ln|x+1|-\ln|x-1|}_{\ln\left|\frac{x+1}{x-1}\right|}+K}$$

Beste modu batera

$$\int \ln(x^2-1)\,dx = \int \ln\left[(x+1)(x-1)\right] dx =$$

$$= \underbrace{\int \underbrace{\ln(x+1)}_{L}\, \underbrace{dx}_{P}}_{I_1} + \underbrace{\int \ln(x-1)\,dx}_{I_2}$$

$I_1)$
$$\begin{cases} u = \ln(x+1) \longrightarrow du = \dfrac{1}{x+1} \\ dv = dx \longrightarrow v = x. \end{cases}$$

$$\begin{array}{r|l} x & x+1 \\ \cline{2-2} -x-1 & 1 \\ \hline -1 & \end{array}$$

$$I_1 = \ln(x+1)\cdot x - \int x \cdot \frac{1}{x+1}\,dx$$

$$= \ln(x+1)\cdot x - \int \left(1 - \frac{1}{x+1}\right) dx =$$

$$= \boxed{\ln(x+1)\cdot x - x + \ln|x+1|} + k$$

$$\begin{array}{r|l} x & x-1 \\ \cline{2-2} +1 & 1 \end{array}$$

$I_2)$
$$I_2 = \ln(x-1)\cdot x - \int x\, \frac{1}{x-1}\,dx$$

$$= \ln(x-1)\cdot x - \int \left(1 + \frac{1}{x-1}\right) dx$$

$$I_2 = \boxed{\ln(x-1)\cdot x - x - \ln|x-1|} + k.$$

$$I = \underbrace{\ln(x+1)\cdot x} - x + \ln|x+1| + \underbrace{\ln(x-1)\cdot x} - x - \ln|x-1| + k$$

$$= \boxed{x\Big(\underbrace{\ln(x+1)\cdot\ln(x-1)}_{x\cdot\ln(x^2-1)}\Big) - 2x + \underbrace{\ln|x+1| - \ln|x-1|}_{\ln\left|\frac{x+1}{x-1}\right|} + k}$$

2021-OHIKOA

$\boxed{B4}$ $I = \int \underbrace{(x+2)}_{P} \cdot \underbrace{\sin(2x)}_{S}\, dx$

Teoria

Biderketa baten deribatua kalkulatzeko formula:

$$D(u(x)\cdot v(x)) = u'(x)\, v(x) + u(x)\, v'(x)$$

Idazkera diferentzialean:

$$d(u(x)\cdot v(x)) = du(x)\cdot v(x) + \underbrace{u(x)\cdot dv(x)}$$

Bakandu+ azkenengo gaia

$$u(x)\cdot dv(x) = d(u(x)\cdot v(x)) - v(x)\, du(x)$$

Bi aldetan integratuz:

$$\boxed{\int u(x)\, dv(x) = u(x)\, v(x) - \int v(x)\cdot du(x)}$$

Integrala $\underline{\text{ZATIKAKO METODOAREKIN}}$ ebatziko dugu:

$$\begin{cases} u = x+2 \longrightarrow du = dx \\ dv = \sin 2x\, dx \longrightarrow v = \int \sin 2x\, dx = -\frac{1}{2}\cos(2x) \end{cases}$$

$$I = (x+2)\left(\frac{-1}{2}\right)\cos(2x) + \int \frac{-1}{2}\cos(2x)\cdot dx.$$

$$\boxed{I = \frac{-1}{2}(x+2)\cos(2x) + \frac{1}{4}\sin(2x)} + K.$$

2021 - OHIKOA

(B4) 2. atala

$$J=\int \frac{x+7}{x^2-4x+5}\,dx = \int \frac{x+7}{(x-5)(x+1)}\,dx =$$

$$\frac{x+7}{x^2-4x+5} = \frac{A}{x-5} + \frac{B}{x+1} = \frac{A(x+1)+B(x-5)}{(x-5)(x+1)}$$

$$x+7 = A(x+1) + B(x-5)$$

$$x=-1 \longrightarrow -1+7 = B(-1-5) \longrightarrow 6 = B(-6) \quad \boxed{B=-1}$$

$$x=5 \longrightarrow 5+7 = A\cdot 6 \longrightarrow \boxed{A=2}$$

$$J=\int \frac{x+7}{x^2-4x+5} = \int \frac{2}{x-5}\,dx + \int \frac{-1}{x+1}\,dx =$$

$$= \boxed{2\ln|x-5| - \ln|x+1| + K}$$

2021- EZ OHIKOA

$\boxed{B4}$ $\int x \cdot \ln(x+1)\,dx$ eta ataldu.

ZATIKAKO METODOA erabiliz ebazten da:

→ Zatikako metodoa ozoltzeko biderkatoren formula aplikatzen da.

$$D(u(x)\cdot v(x)) = u'(x)\cdot v(x) + u(x)\cdot v'(x).$$

→ Diferentzial eran → $d[u(x)\cdot v(x)] = du(x)\cdot v(x) + \underbrace{u(x)dv(x)}$

→ Azken gaia bakandu: $u(x)\,dv(x) = d[u(x)\cdot v(x)] - v(x)\cdot du(x)$.

→ Integratzen $\int u(x)\cdot dv(x) = u(x)\cdot v(x) - \int v(x)\,du(x)$

Beraz $\int u(x)\,dv(x)$ formako integralak ebazteko $\int v(x)\,du(x)$ -tik abiatzen gara.

$$I = \int \underset{P}{x}\cdot \underset{L}{\underline{\ln(x+1)}}\,dx \qquad \begin{cases} u = \ln(x+1) \longrightarrow du = \dfrac{1}{x+1}\,dx. \\ dv = x \longrightarrow v = \dfrac{x^2}{2} \end{cases}$$

$$I = \ln(x+1)\cdot\frac{x^2}{2} - \int \frac{x^2}{2}\cdot\frac{1}{x+1}\,dx =$$

$$I = \frac{x^2}{2}\cdot\ln(x+1) - \frac{1}{2}\int\frac{x^2}{x+1}\,dx.$$

$$\begin{array}{r|l} x^2 & x+1 \\ \cline{2-2} -x^2-x & x-1 \\ \hline -x & \\ +x+1 & \\ \hline 1 & \end{array}$$

$$I = \frac{x^2}{2}\cdot\ln(x+1) - \frac{1}{2}\int\left(x-1+\frac{1}{x+1}\right)dx$$

$$\boxed{I = \frac{x^2}{2}\ln(x+1) - \frac{x^2}{4} + \frac{x}{2} - \frac{1}{2}\ln|x+1| + K}$$

2020 - OHIKOA

$\boxed{B4}$ $I = \int \underbrace{x}_{P} \cdot \underbrace{\cos(2x)}_{S.} \, dx$

- Lehenengo partea metodoa atotkeo da ______.
- Biporren partea ebatzi:

ZATIKAKO METODOA $\int u \, dv = u \cdot v - \int v \cdot du.$

$$\begin{cases} u = x \longrightarrow du = dx \\ dv = \cos(2x)dx \longrightarrow v = \int \cos(2x)\,dx = \frac{1}{2}\int 2\cos(2x)\,dx \end{cases}$$

$$v = \frac{1}{2}\sin(2x).$$

$$I = x \cdot \frac{1}{2}\sin(2x) - \int \frac{1}{2}\sin(2x) \cdot dx =$$

$$= \frac{x \cdot \sin(2x)}{2} - \frac{1}{2}\frac{1}{2}\int 2\sin(2x) \cdot dx =$$

$$= \frac{x \cdot \sin(2x)}{2} + \frac{1}{4}\cos(2x). \; + K$$

$$J = \int \frac{dx}{x^2+2x-3} = \int \frac{dx}{(x+3)(x-1)}$$

$$\frac{1}{x^2+2x-3} = \frac{1}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1} = \frac{A(x-1)+B(x+3)}{(x+3)(x-1)}$$

$$1 = A(x-1) + B(x+3)$$

$x=1 \quad 1 = B \cdot 4 \longrightarrow \boxed{B = 1/4}$

$x=-3 \quad 1 = A \cdot (-4) \longrightarrow \boxed{A = -1/4}$

$$J = \int \frac{dx}{(x+3)(x-1)} = \int \frac{-1/4\,dx}{x+3} + \int \frac{1/4}{x-1}dx =$$

$$\boxed{J = -\frac{1}{4}\ln|x+3| + \frac{1}{4}\cdot\ln|x-1| + k}$$

2020 - EZ OHIKOA

B4 $\displaystyle\int \underset{P}{x} \cdot \underset{S}{\cos(3x)} \cdot dx.$

ZATIKAKO METODOA

$u = x \longrightarrow du = dx$

$dv = \cos(3x)\,dx$

$v = \frac{1}{3}\int 3\cos(3x)\,dx = \frac{1}{3}\sin(3x)$

$$I = x \cdot \frac{1}{3}\sin(3x) - \int \frac{1}{3}\sin(3x)\cdot dx$$

$$= \frac{x \cdot \sin(3x)}{3} - \frac{1}{3}\,\frac{1}{3}\int 3\sin(3x)\,dx =$$

$$= \frac{x \cdot \sin(3x)}{3} - \frac{1}{9}\left(-\cos(3x)\right) + K$$

$$\boxed{I = \frac{x \cdot \sin(3x)}{3} + \frac{\cos(3x)}{9} + K}$$

EBAU - 2019 - EKAINA

A4 $\displaystyle\int \underset{P}{x} \cdot \underset{E}{e^{-4x}}\, dx$

ZATIKAKO METODOA

$\int u \cdot dv = u \cdot v - \int v \cdot du$

$$\begin{cases} u = x \longrightarrow du = dx \\ dv = e^{-4x} \longrightarrow v = \frac{1}{-4}\int -4e^{-4x}\,dx \end{cases}$$

$$v = -\frac{1}{4}e^{-4x}$$

$$I = x \cdot \left(-\frac{1}{4}\right)e^{-4x} - \int -\frac{1}{4}e^{-4x}\,dx = -\frac{1}{4}xe^{-4x} + \frac{1}{4(-4)}\int (-4)e^{-4x}\,dx$$

$$= -\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x} + k.$$

$$\boxed{I = -\frac{1}{16}e^{-4x}(4x-1) + k}$$

EBAU. 2019. - UZTAILA

$\boxed{B4}$ $\displaystyle\int \frac{8x+7}{(x+1)(x+3)}$

Deskonposatzen da zatiki aljebraikoa, zatiki sinpleetan:

$$\frac{8x+7}{(x+1)(x+3)} = \frac{A}{x+1} + \frac{B}{x+3} = \frac{A(x+3)+B(x+1)}{(x+1)(x+3)}$$

$$8x+7 = A(x+3) + B(x+1)$$

$$x=-3 \longrightarrow -24+7 = \cancel{A\cdot 0} + B(-2)$$

$$-17 = -2B \longrightarrow \boxed{B = 17/2}$$

$$x=-1 \longrightarrow -8+7 = A(-1+3) + \cancel{B\cdot 0}$$

$$-1 = 2A \longrightarrow \boxed{A = -1/2}$$

$$I = \int \frac{8x+7}{(x+1)(x+3)} = \int \frac{-1/2}{x+1}\,dx + \int \frac{17/2}{x+3}\,dx =$$

$$= \boxed{-\frac{1}{2}\ln|x+1| + \frac{17}{2}\ln|x+3| + K}$$

EBAU 2018 · EKAINA

$\boxed{A4}$ $\displaystyle\int \frac{2x-1}{x(x+1)^2}\,dx$

$$\frac{2x-1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2} = \frac{A(x+1)^2 + B(x+1)x + Cx}{x\cdot(x+1)^2}$$

$$2x-1 = A(x+1)^2 + B(x+1)x + Cx$$

$x=0 \longrightarrow \quad -1 = A + \cancel{B\cdot 0} + \cancel{C\cdot 0} \qquad \boxed{A=-1}$

$x=-1 \longrightarrow \quad -3 = \cancel{A\cdot 0} + \cancel{B\cdot 0} - C \qquad \boxed{C=3}$

$x=1 \longrightarrow \quad 1 = 4A + 2B + C$

$1 = -4 + 2B + 3 \longrightarrow \boxed{B=1}$

$$I = \int\left(\frac{-1}{x} + \frac{1}{x+1} + \frac{3}{(x+1)^2}\right)dx =$$

$$= -\ln|x| + \ln|x+1| + 3\cdot\frac{(x+1)^{-2+1}}{-2+1} + k$$

$$= \boxed{\ln\left|\frac{x+1}{x}\right| - \frac{3}{x+1} + k}$$

EBAU 2018 - UZTAILA

A4 $\quad I = \int \underset{P}{x^2} \cdot \underset{E}{e^{-3x}}\, dx$ $\qquad$ ZATIKAKO METODOA

$$I \left\{ \begin{array}{l} u = x^2 \longrightarrow du = 2x\,dx \\ dv = e^{-3x} \longrightarrow v = \int e^{-3x}\,dx = -\frac{1}{3}e^{-3x} \end{array} \right.$$

$$I = x^2\cdot\left(-\frac{1}{3}e^{-3x}\right) - \int -\frac{1}{3}e^{-3x}\cdot 2x\,dx =$$

$$= -\frac{x^2}{3}e^{-3x} + \frac{2}{3}\underbrace{\int \underset{P}{x}\cdot \underset{E}{e^{-3x}}\,dx}_{I_1}$$

$$I_1 \left| \begin{array}{l} u = x \longrightarrow du = dx \\ dv = e^{-3x} \longrightarrow v = \int e^{-3x}\,dx = -\frac{1}{3}e^{-3x} \end{array} \right.$$

$$I = -\frac{x^2}{3}e^{-3x} + \frac{2}{3}I_1$$

$$I = -\frac{x^2}{3}e^{-3x} + \frac{2}{3}\left[x\cdot\left(-\frac{1}{3}\right)e^{-3x} - \int -\frac{1}{3}e^{-3x}\cdot dx\right] =$$

$$I = -\frac{x^2}{3}e^{-3x} - \frac{2}{9}x\,e^{-3x} + \frac{2}{9}\left(-\frac{1}{3}e^{-3x}\right) + K$$

$$I = \frac{e^{-3x}}{3}\left(-x^2 - \frac{2}{3}x - \frac{2}{9}\right) + K.$$